Problem Set #15

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1 Problem Description

An RLC circuit contains a resistor, inductor, and capacitor, connected in series or parallel. The circuit is a harmonic oscillator of current, and the current oscillation maximized when the frequency gets closer to the resonance frequency. The object of the problem is to build and measure the performance of a simple RLC oscillator. We measure the oscillations as a function of time and compare the experimental results to what is theoretical expected.

2 Material

- 1. ELVIS breadboard
- 2. ELVIS oscilloscope
- 3. ELVIS function generator
- 4. Jumper cables
- 5. 1000 K Ω resistor
- 6. 10 nF capacitor
- 7. 10 mH inductor

3 Procedure

- 1. Set up linear RLC circuit and software
- 2. Obtain resonance frequency using $1/2\pi(LC)$
- 3. Measure time vs current graph for resistor

RLC circuit has been set up by using 1000 K Ω resistor, 10 mH inductor, and a 10 nF capacitor. Each end of the circuit is connected to grounds and, a sinosoidal voltage supplier with 1-V peak to peak. The supply voltage and voltage difference between the 1000 K Ω resistor has been measured. The accuracy and range of the voltage measurement of the NI ELVIS device is $0.3\% \pm 0.001\%$ and the range is $\pm 14|V|rms$

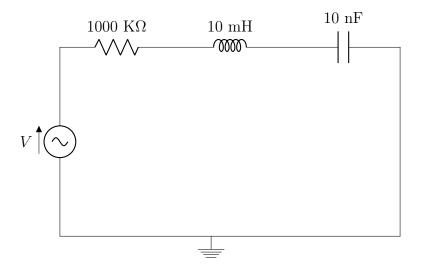


Figure 1: Diagram of the RLC circuit.

4 Result

The circuit has been analyzed by phasor form. The current analyze of the RLC circuit can be represented by following equation.

$$I_0 = \frac{V_0}{\sqrt{R^2 + (\omega L + \frac{1}{\omega C})^2}}$$
(1)

Since values of R, L, C, I_0 , and V_0 are known, RLC circuit which has been used in the experiment can be explained by using following equation.

$$I_0 = \frac{2}{\sqrt{(10 \text{ K}\Omega)^2 + (\omega 10 \text{ mH} + \frac{1}{\omega 10 \text{ nF}})^2}}$$
(2)

The total impedance of the series RLC circuit is

$$\omega = 2\pi f$$

$$R + L + C = R + j\omega L + \frac{1}{j\omega C}$$
(3)

The circuit shows resonance behavior when the impedance of the Inductor and Capacitor is minimized, because the current goes directly through the Resistor.

Frequency(KHz)	lp-p(mA)
1	0.00006204
2	0.00012133
3	0.00018367
4	0.000246
5	0.00031256
6	0.00037664
7	0.00044485
8	0.00051481
9	0.0005847
10	0.00065117
11	0.00071222
12	0.00077114
13	0.00082265
14	0.00086093
15	0.00088767
16	0.00090117
16.2	0.00090244
17	0.00090156
18	0.00089373
<mark>1</mark> 9	0.00087702
20	0.00085352
21	0.00082712
22	0.00079888
23	0.00076935
24	0.00074016
25	0.0007052
26	0.00068484
27	0.00065737
28	0.00063134
29	0.00060687
30	0.00058638
31	0.00056346

Figure 2: Data collected from the experiment.

$$\omega = 2\pi f$$

$$R + L + C = R + j\omega L + \frac{1}{j\omega C}$$
(4)

When the impedance of L and C is minimized is

$$j\omega L + \frac{1}{j\omega C} = 0$$

$$j(\omega L + \frac{1}{-\omega C}) = 0$$

$$\omega L + \frac{1}{-\omega C} = 0$$

$$\omega^{2} = \frac{1}{LC}$$
(5)

Since the $\omega = 2\pi f$,

$$(2\pi f)^2 = \frac{1}{LC}$$

$$4\pi^2 f^2 = \frac{1}{LC}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$
(6)

Therefore, expected resonance frequency of the circuit is

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10^{(-2)} * 10^{(-8)}}} = 16 \text{ KHz}$$
(7)

MATLAB code was used to plot simulation and experiment result. The following set of code is used to create a graph, and Figure 4 is the result of the MATLAB code.

```
1

2

3 V0 = 1;

4 R = 1000000;

5 L = 10^{(-2)};

6 C = 10^{(-8)};

7 I = @(f) V0./ sqrt(R+(f*2*pi*L-1./(f*2*pi*C)).^2);

8
```

```
9 fe = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 16.2 & 17 & 18 \end{bmatrix}
       19 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25 \ 26 \ 27 \ 28 \ 29 \ 30 \ 31];
10 \text{ Vpp} = [62.04 \ 121.33 \ 183.67 \ 246.00 \ 312.56 \ 376.64 \ 444.85]
      514.81 584.70 651.17 712.22 771.14 822.65 860.93
      887.67 \ 901.17 \ 902.44 \ 901.56 \ 893.73 \ 877.02 \ 853.52
      827.12 798.88 769.35 740.16 705.2 684.84 657.37
      631.34 606.87 586.38 563.46];
11
12 plot (fe, Vpp./R, fe, I (fe *1000), 'Linewidth', 2)
13 legend ('ExpData', 'Simul')
14 xlim ([2 \ 30]);
15 ylim ([-0.00001 \ 0.0011]);
16 ylabel('Current (mA)');
17 xlabel('Frequency (KHz)');
18 title ('Frequency-Current from Experiment and Simulation
      ');
```

19 grid on

The measured resonance frequency is 16.2 kHz, therefore, the resonance frequency matches the expectation. The data from experiment shows the expected graph. The maximum voltage is measured at the frequency of 16.2 kHz and the value is 902.44 mV.

As shown in the Figure 5, the error between experimental data and simulation data tends to be larger as it goes to resonance frequency. The source of the error is unknown, but is consistent with some nonlinearity in the current measurement.

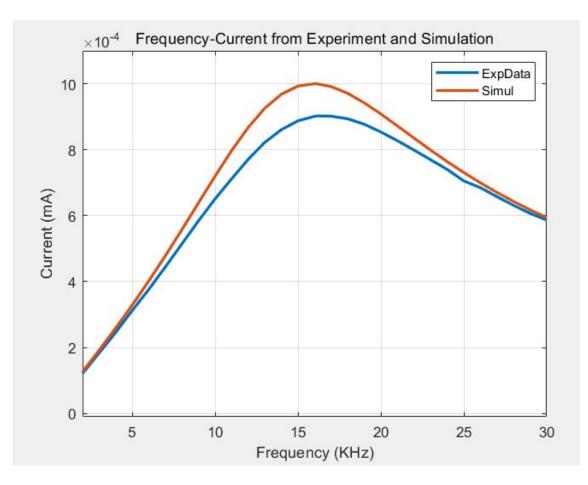


Figure 3: Plot of the output current across the 1000 $k\Omega$ resistor, with Experimental data and Simulation data.

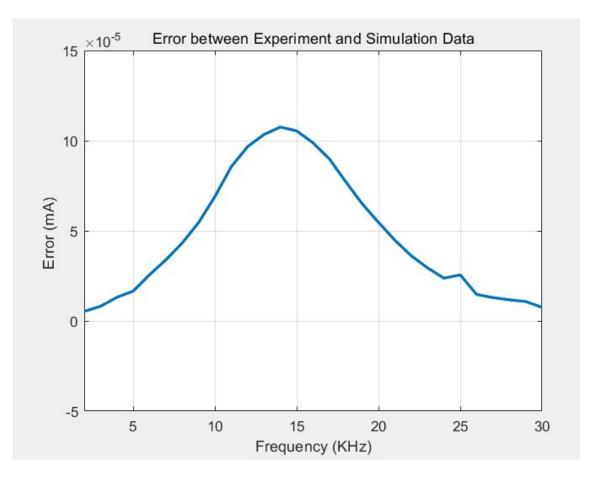


Figure 4: Error between simulation and experiment data.